Contents

1 Introduction 1

1.1. Numerical Analysis in Engineering 1
- 1.1.1. Decision Making in Engineering 2
- 1.1.2. Expected Educational Outcomes 3

1.2. Analytical Versus Numerical Analysis 4

1.3. Taylor Series Expansion 4

1.4. Applications 8
- 1.4.1. Taylor Series Expansion of the Square Root 8
- 1.4.2. Example Taylor Series 9

1.5. Problems 10

2 Matrices 11

2.1. Introduction 11
- 2.1.1. Definition of a Matrix 11
- 2.1.2. Formation of a Matrix 12
- 2.1.3. Types of Matrices 14

2.2. Matrix Operations 15
- 2.2.1. Matrix Equality 15
- 2.2.2. Matrix Addition and Subtraction 15
- 2.2.3. Matrix Multiplication: An Introductory Example 16
- 2.2.4. Matrix Multiplication: General Rules 17
- 2.2.5. Matrix Multiplication by a Scalar 19
- 2.2.6. Matrix Inversion 19
- 2.2.7. Matrix Singularity 20
- 2.2.8. Additional Topics in Matrix Algebra 20

2.3. Vectors 22
- 2.3.1. Definitions 22
- 2.3.2. Vector Operations 23
- 2.3.3. Orthogonal and Normalized Vectors 23

2.4. Determinants 24

2.5. Rank of a Matrix 28
2.6. Applications 29
2.6.1. Reactions of a Beam Due to Loads 29
2.6.2. Correlation Analysis of Water Quality Data 32
2.6.3. Regression Analysis for Predicting Bus Ridership 34
2.7. Problems 35

3 Introduction to Numerical Methods 40

3.1. Introduction 40
3.1.1. Characteristics of Numerical Methods 41
3.2. Accuracy, Precision, and Bias 43
3.3. Significant Figures 44
3.4. Analysis of Numerical Errors 47
3.4.1. Error Types 47
3.4.2. Measurement and Truncation Errors 49
3.4.3. Error Analysis in Numerical Solutions 50
3.5. Advantages and Disadvantages of Numerical Methods 52
3.6. Applications 53
3.6.1. Pipe Design 53
3.6.2. Bending Moment for a Beam 55
3.7. Problems 59

4 Roots of Equations 61

4.1. Introduction 61
4.2. Eigenvalue Analysis 62
4.3. Direct-search Method 64
4.4. Bisection Method 68
4.4.1. Error Analysis and Convergence Criterion 69
4.5. Newton–Raphson Iteration 72
4.5.1. Nonconvergence 75
4.6. Secant Method 77
4.7. Polynomial Reduction 79
4.8. Synthetic Division 80
4.8.1. Programming Considerations 81
4.9. Multiple Roots 85
4.10. Systems of Nonlinear Equations 86
4.11. Applications 90
4.11.1. Pipe Flow Evaluation 90
4.11.2. Gas Law 91

4.12. Problems 92

5 Simultaneous Linear Equations 96

5.1. Introduction 96
- 5.1.1. General Form for a System of Equations 98
- 5.1.2. Solution of Two Equations 99
- 5.1.3. Classification of Systems of Equations Based on Graphical Interpretation 100

5.2. Gaussian Elimination 102
- 5.2.1. Permissible Operations 102
- 5.2.2. Matrix Representation of the System of Equations 103
- 5.2.3. Gaussian Elimination Procedure 104

5.3. Gauss–Jordan Elimination 109

5.4. Additional Considerations for Elimination Procedures 111
- 5.4.1. Accumulated Round-off Errors 111
- 5.4.2. Zero Pivot Element 112
- 5.4.3. Considerations in Programming 112

5.5. LU Decomposition 113
- 5.5.1. General Case 113
- 5.5.2. Banded Matrices 117
- 5.5.3. Symmetric Matrices 118

5.6. Iterative Equation-solving Methods 119
- 5.6.1. Jacobi Iteration 120
- 5.6.2. Gaussian–Seidel Iteration 122
- 5.6.3. Convergence Considerations of the Iterative Methods 124
- 5.6.4. Considerations in Programming 127

5.7. Use of Determinants 130
- 5.7.1. Considerations in Programming 133

5.8. Matrix Inversion 133

5.9. Applications 136
- 5.9.1. Flexibility and Stiffness Analyses of a Beam 136
- 5.9.2. Concrete Mix Design 138

5.10. Problems 140

6 Numerical Interpolation 146

6.1. Introduction 146

6.2. Method of Undetermined Coefficients 146

6.3. Gregory–Newton Interpolation Method 149

6.4. Finite-difference Interpolation 152
- 6.4.1. Finite-difference Table 155

6.5. Newton's Method 157

6.6. Lagrange Polynomials 161
6.7. Interpolation Using Splines 163
6.7.1. Linear Splines 163
6.7.2. Quadratic Splines 165
6.7.3. Cubic Splines and Other Higher-order Splines 167
6.8. Guidelines for Choice of Interpolation Method 168
6.9. Multidimensional Interpolation 169
6.9.1. Linear Interpolation in Two Dimensions 170
6.10. Applications 171
6.10.1. Probability of Wind Loading 171
6.10.2. Shear Stress of Oil Between Two Parallel Plates 173
6.11. Problems 174

7 Differentiation and Integration 180

7.1. Numerical Differentiation 180
7.1.1. Finite-difference Differentiation 180
7.1.2. Differentiation Using a Finite-difference Table 184
7.1.3. Differentiating an Interpolating Polynomial 186
7.1.4. Differentiation Using Taylor Series Expansion 187
7.2. Numerical Integration 189
7.2.1. Interpolation Formula Approach 190
7.2.2. Trapezoidal Rule 192
7.2.3. Simpson's Rule 197
7.2.4. Romberg Integration 198
7.3. Applications 201
7.3.1. Estimating Areas 201
7.3.2. Pipe Flow Rate 202
7.3.3. Volume of Gabion Retaining Wall 204
7.3.4. Sediment Loads in Rivers 205
7.3.5. Probability Computations Using the Standard Normal Probability Distribution 205
7.4. Problems 206

8 Differential Equations 212

8.1. Introduction 212
8.1.1. Definitions 212
8.1.2. Origin of Differential Equations 213
8.2. Taylor Series Expansion 214
8.2.1. Fundamental Case 214
8.2.2. General Case 216
8.3. Euler's Method 217
8.3.1. Errors with Euler's Method 218
8.4. Modified Euler's Method 221

8.5. Runge–Kutta Methods 223
8.5.1. Second-order Runge–Kutta Methods 223
8.5.2. Third-order Runge–Kutta Methods 224
8.5.3. Fourth-order Runge–Kutta Methods 225
8.6. Predictor–corrector Methods 227
8.6.1. Euler–trapezoidal Method 228
8.6.2. Milne–Simpson Method 229
8.7. Least-squares Method 231
8.8. Galerkin Method 235
8.9. Higher-order Differential Equations 237
8.10. Boundary-value Problems 240
8.10.1. Shooting Method 241
8.10.2. Finite-difference Methods 241
8.11. Integral Equations 242
8.12. Applications 242
8.12.1. Motion of a Falling Body 242
8.12.2. Electrical Circuit 243
8.12.3. One-dimensional Heat Flow 245
8.12.4. Estimating Biochemical Oxygen Demand 246
8.12.5. Accumulation of Eroded Soil in a Sediment Trap 248
8.12.6. Growth Rate of Bacteria 250
8.12.7. Bending Moment and Shear Force for a Beam 251
8.12.8. Dynamic Response of a Structure 253
8.12.9. Deflection of a Beam 259
8.13. Problems 262

9 Data Description and Treatment 267

9.1. Introduction 267
9.2. Classification of Data 267
9.2.1. Nominal Scale 268
9.2.2. Ordinal Scale 268
9.2.3. Interval Scale 268
9.2.4. Ratio Scale 269
9.2.5. Dimensionality of Data 269
9.3. Graphical Description of Data 269
9.3.1. Area Charts 270
9.3.2. Pie Charts 271
9.3.3. Bar Charts 271
9.3.4. Column Charts 272
9.3.5. Scatter Diagrams 273
9.3.6. Line Graphs 273
9.3.7. Combination Charts 275
9.3.8. Three-dimensional Charts 276

9.4. Histograms and Frequency Diagrams 278
9.5. Descriptive Measures 279
9.5.1. Central Tendency Measures 279
9.5.2. Dispersion Measures 280
9.5.3. Percentiles 282
9.5.4. Box-and-whisker Plots 283
9.6. Applications 284
9.6.1. Two Random Samples 284
9.6.2. Stage and Discharge of a River 286
9.7. Problems 289

10 Curve Fitting and Regression Analysis 293

10.1. Introduction 293
10.2. Correlation Analysis 293
10.2.1. Graphical Analysis 294
10.2.2. Bivariate Correlation 297
10.2.3. Separation of Variation 297
10.2.4. Correlation: Fraction of Explained Variation 298
10.2.5. Computational Form of Correlation Coefficient 299
10.3. Introduction to Regression 300
10.3.1. Elements of Statistical Optimization 301
10.3.2. Zero-intercept Model 302
10.3.3. Regression Definitions 303
10.4. Principle of Least Squares 304
10.4.1. Definitions 304
10.4.2. Solution Procedure 304
10.5. Reliability of the Regression Equation 306
10.5.1. Correlation Coefficient 306
10.5.2. Standard Error of Estimate 307
10.5.3. Standardized Partial Regression Coefficients 308
10.5.4. Assumptions Underlying the Regression Model 309
10.6. Correlation Versus Regression 311
10.7. Applications of Bivariate Regression Analysis 312
10.7.1. Estimating Trip Rate 312
10.7.2. Breakwater Cost 312
10.7.3. Stress–strain Analysis 314
10.7.4. Project Cost versus Time 314
10.7.5. Effect of Extreme Event 317
10.7.6. Variable Transformation 318
10.8. Multiple Regression Analysis 319
10.8.1. Correlation Matrix 320
10.8.2. Calibration of the Multiple Linear Model 322
10.8.3. Standardized Model 323

10.8.4. Intercorrelation 324
10.8.5. Criteria for Evaluating a Multiple Regression Model 326
10.8.6. Analysis of Residuals 327
10.8.7. Computational Example 327
10.9. Regression Analysis of Nonlinear Models 330
10.9.1. Common Nonlinear Alternatives 330
10.9.2. Calibration of Polynomial Models 331
10.9.3. Fitting a Power Model 332
10.9.4. Goodness of Fit 333
10.9.5. Additional Model Forms 334
10.10. Applications 334
10.10.1. One-predictor Polynomial of Sediment Yield versus Slope 334
10.10.2. One-predictor Polynomial of Evaporation versus Temperature 335
10.10.3. Single-predictor Power Model 336
10.10.4. Multivariate Power Model 337
10.10.5. Estimating Breakwater Costs 339
10.10.6. Trip Generation Model 341
10.10.7. Estimation of the Reaeration Coefficient 342
10.10.8. Estimating Slope Stability 343
10.11. Problems 347

Index 353