Contents

	PREFACE			
	ACKN	ACKNOWLEDGMENTS		
0	INTRODUCTION AND BACKGROUND			
	0.1	Qualitative Systems Concepts 3		
		0.1.1 Qualitative Structures Concepts, 4		
	0.2	Qualitative Probability Concepts 4		
	0.3	Qualitative Vibration, Uncertainties, and Control 5		
	0.4	Analysis and Design: Interrelationships 5		
	0.5	.5 Computational Modeling and Design 6		
	0.6	Modeling for Vibration 6		
		 0.6.1 Problem Idealization and Formulation, 6 0.6.2 Concepts of Stiffness, Damping, and Inertia, 10 0.6.3 Statics and Equilibrium, 12 0.6.4 The Equations of Motion, 12 		

16

		0.6.5 Types of System Models, 12	
	0.7	Units 14	
1	SON	NE MATHEMATICS	16
	1.1	Taylor Series and Linearization 16	
	1.2	Ordinary Differential Equations 18	
		 1.2.1 Solution of Linear Equations, 19 1.2.2 Homogeneous Solution, 20 1.2.3 Particular Solution, 24 	
	1.3	Matrices 26	
		 1.3.1 Matrix Operations, 27 1.3.2 Determinant and Matrix Inverse, 28 1.3.3 Eigenvalues and Eigenvectors of a Square Matrix, 30 	
	1.4	Transition 31	
?	SING TO D	Example Problems and Motivation 33	32
		2.1.1 Transport of a Satellite, 332.1.2 Rocket Ship, 33	
	2.2	Mathematical Modeling: Deterministic 37	
		 2.2.1 Problem Idealization and Formulation, 37 2.2.2 Mass, Damping, and Stiffness, 39 2.2.3 Sources of Deterministic Approximation, 40 2.2.4 Dimensional Analysis, 41 2.2.5 Equations of Motion: Newton's Second Law, 43 2.2.6 Equations of Motion: Energy Formulation, 49 2.2.7 The Rotating Vector Approach to the Equation of Motion, 53 2.2.8 Solution of the Equations of Motion, 54 	
	2.3	Free Vibration with No Damping 55	
		2.3.1 Alternate Formulation, 58	
	2.4	Harmonic Forced Vibration with No Damping 59	
		2.4.1 Resonance, 602.4.2 Vibration of a Structure in Water, 65	
	2.5	Concepts Summary 66	
	2.6	Problems 67	

Contents

3	SINGLE DEGREE OF FREEDOM VIBRATION: DISCRETE MODELS WITH DAMPING 7		
,	3.1	Free Vibration with Damping 74	
		3.1.1 Some Time Constants, 81	
	3.2	Forced Vibration with Damping 81	
	3.3	Harmonic Excitation and Damped Response 84	
		3.3.1 Harmonic Excitation in Complex Notation, 90	
	3.4	Periodic but Not Harmonic Excitation 103	
	3.5	Arbitrary Loading: Laplace Transform 106	
	3.6	Step Loading 113	
	3.7	Impulsive Excitation 117	
	3.8	Arbitrary Loading: Convolution 119	
	3.9	Introduction to the Use of Lagrange's Equation 124	
	3.10	Notions of Randomness 127	
	3.11	Notions of Control 128	
	3.12	The Inverse Problem 129	
	3.13	A Self-Excited System and Stability 129	
	3.14	Solution Analysis and Design Techniques 130	
	3.15	Concepts Summary 140	
	3.16	Problems 140	
	3.17	Mini-Projects 151	
4 SINGLE DEGREE OF FREEDOM VIBRATION: PROBABILISTIC MODELS OF FORCES			154
	4.1	Introduction 154	
	4.2	Example Problems and Motivation 158	
		 4.2.1 Random Vibration, 158 4.2.2 Fatigue Life, 159 4.2.3 Ocean Wave Forces, 160 4.2.4 Wind Forces, 162 4.2.5 Material Properties, 163 4.2.6 Statistical Information: Link to Probabilistic Modeling, 164 	

5

4.3 Random Variables 164			
	4.3.1 4.3.2	Probability Distribution, 165 Probability Density Function, 166	
4.4	Mathe	ematical Expectation 168	
	4.4.1	Variance, 169	
4.5	Proba	bility Densities Useful in Applications 170	
	4.5.1 4.5.2 4.5.3 4.5.4 4.5.5	The Uniform Density, 170 The Exponential Density, 173 The Normal (Gaussian) Density, 173 The Lognormal Density, 176 The Rayleigh Density, 176	
4.6	Two F	Random Variables 177	
	4.6.1	Covariance and Correlation, 179	
4.7	Rando	om Processes 183	
	4.7.1 4.7.2 4.7.3 4.7.4	Basic Random Process Descriptors, 184 Ensemble Averaging, 184 Stationarity, 188 Power Spectrum, 189	
4.8	Rando	om Vibration 196	
		Formulation, 196 Derivation of Equations, 197 Response Correlations, 197 Response Spectral Densities, 199'	
4.9	Conce	pts Summary 201	
4.10	Proble	ms 202	
4.11	Mini-P	Projects 207	
SINGL	E DEG	REE OF FREEDOM VIBRATION: CONTROL	209
5.1	Introd	uction 210	
5.2	Feedback Control 211		
5.3	Perform	mance of Feedback Control Systems 214	
	5.3.2	Poles and Zeros of a Second Order System, 216 Gain Factor, 217 Stability of Response, 218	
5.4	Autom	natic Control of Transient Response 219	
		Control Actions, 219	

		5.4.2 Control of Transient Response, 221	
	5.5	Sensitivity to Parameter Variations 225	
	5.6	State Variable Models 228	
		5.6.1 Matrix Derivatives and Integrals, 231	
	5.7	Concepts Summary 232	
	5.8	Problems 232	
	5.9	Mini-Projects 235	
6	PRINC	CIPLE OF VIRTUAL WORK AND LAGRANGE'S EQUATION	237
	6.1	Introduction 237	
	6.2	Virtual Work 239	
		6.2.1 Work and Energy, 2396.2.2 Virtual Work, 2426.2.3 D'Alembert's Principle, 244	
	6.3	Lagrange's Equation 248	
		6.3.1 Lagrange's Equation for Small Oscillations, 255	
•	6.4	Hamilton's Principle 256	
	6.5	Lagrange's Equation with Damping 260	
	6.6	Concepts Summary 261	
	6.7	Problems 262	
	6.8	Mini-Projects 265	
7	MULT	I DEGREE OF FREEDOM VIBRATION	267
	7.1	Example Problems and Motivation 267	
		7.1.1 Periodic Structures, 2687.1.2 Inverse Problems, 268	
	7.2	Preliminary Concepts of Stiffness and Flexibility 268	
		7.2.1 Influence Coefficients, 269	
	7.3	Derivation of Equations of Motion 275	
	*	7.3.1 [m] and [k] Matrix Properties, 281	
٠.	7.4	Undamped Vibration 282	
		7.4.1 Two Degree of Freedom Motion: Solution by the Direct Method, 282	

	7.4.2 Forced Vibration by the Direct Method, 2887.4.3 Coupled Pendula—Beating, 293				
7.5	Free Vibration with Damping: Solution by the Direct Method 29				
7.6	Modal Analysis 308				
	 7.6.1 Orthogonality of Normal Modes, 309 7.6.2 Modal Analysis with Forcing, 314 7.6.3 Modal Analysis with Damping: Proportional Damping, 322 7.6.4 Modal Analysis Compared to the Direct Method, 325 				
7.7	Generalization to n Degrees of Freedom 327				
	7.7.1 Modal Matrix [P], 328				
7.8	Unrestrained Systems 329				
	7.8.1 Repeated Frequencies, 336				
7.9	The Geometry of the Eigenvalue Problem 336				
	7.9.1 Repeated Frequencies, 339				
7.10	Periodic Structures 339				
	7.10.1 Perfect Lattice Models, 3407.10.2 Effects of Imperfection, 342				
7.11	Inverse Vibration: Estimation of Mass and Stiffness 343				
	7.11.1 Deterministic Inverse Vibration Problem, 3457.11.2 Effect of Uncertain Data, 348				
7.12	Sloshing of Fluids in Containers: Equivalent Mechanical Models 352				
7.13	Stability of Motion 355				
7.14	Stochastic Response of a Linear Multi Degree of Freedom System 358				
7.15	Rayleigh's Quotient, An Approximate Method 361				
7.16	Monte Carlo Simulation 366				
	 7.16.1 Random Number Generation, 366 7.16.2 Generation of Random Variates, 368 7.16.3 Generating a Time-History for a Random Process Defined by a Power Spectral Density, 370 				
7.17	Concepts Summary 372				
7.18	Problems 373				
7.19	Mini-Projects 382				

8 CONTINUOUS MODELS FOR VIBRATION

8.1	Contin	uous Limit of a Discrete Formulation 386
8.2	Vibrat	ion of Strings 388
	8.2.1 8.2.2 8.2.3 8.2.4	The Wave Equation via Hamilton's Principle, 390 The Boundary Value Problem for the String, 393 Wave Propagation Solution, 394 Modal Solution for Fixed-Fixed Boundary Conditions, 397
8.3	Longit	udinal (Axial) Vibration of Beams 401
	8.3.1 8.3.2 8.3.3 8.3.4	Newton's Approach to the Governing Equation, 402 Hamilton's Approach to the Governing Equation, 404 Simplified Eigenvalue Problem, 405 Orthogonality of the Normal Modes, 408
8.4	Torsio	nal Vibration of Shafts 410
	8.4.1	Torsion of Shaft with Rigid Disk at End, 412
8.5	Transv	verse Vibration of Beams 413
	8.5.1	Derivation of the Equations of Motion for the Beam with Shear Distortion: The Timoshenko Beam, 414
	8.5.2 8.5.3 8.5.4	Boundary Conditions, 418 A Simplified Eigenvalue Problem, 419 Orthogonality of the Normal Modes, 426
8.6	Beam	Vibration: Special Problems 431
	8.6.1 8.6.2 8.6.3 8.6.4 8.6.5	Transverse Vibration of Beam with Axial Force, 431 Transverse Vibration of Beam with Elastic Supports, 434 Transverse Vibration of Beam on Elastic Foundation, 436 Response of a Beam with a Moving Support, 437 Response of a Beam to a Traveling Force, 439
8.7	Vibrat	tion of Membranes 440
	8.7.1 8.7.2	Rectangular Membranes, 440 Circular Membranes, 446
8.8	Vibra	tion of Plates 452
	8.8.1 8.8.2	Derivation of the Equation of Motion for Rectangular Plates, 452 The Eigenvalue Problem, 456
8.9	Rando	om Vibration of Continuous Structures 459
8.10	Appro	oximate Methods for Continuous Structures 463
	8.10.2	Rayleigh's Quotient, 463 Rayleigh-Ritz Method, 464 The Galerkin Method, 470

8.11	Where Variables Do Not Separate 473
	8.11.1 Response Due to Non-Harmonic, Time-Dependent Boundary Conditions, 474
	8.11.2 Flow in a Pipe with Constant Tension, 482
8.12	Concepts Summary 487
8.13	Problems 488
8.14	Mini-Projects 496

INDEX

507